

# Ireland's Participation in the 64th International Mathematical Olympiad

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The 64<sup>th</sup> International Mathematical Olympiad (IMO) took place in Chiba, Japan, from 2–13 July 2023. A total of 618 students (67 of whom were girls) participated from 112 countries.

The Irish delegation consisted of six students (see Table 1) accompanied by Luke Gardiner (Deputy Leader, University of Cambridge, UK) and Mark Flanagan (Team Leader, UCD, Ireland).

## 1 Team selection and preparation

The team detailed in Table 1 consisted of those six students (in order) who scored highest in the Irish Mathematical Olympiad (IrMO), which was held for the 36<sup>th</sup> time on Saturday, 6<sup>th</sup> May, 2023. The IrMO contest consists of two 3-hour papers on one day with five problems on each paper. The students who participated in the IrMO sat the exam simultaneously in one of six *Mathematics Enrichment Centres* (University College Cork, University College Dublin, University of Galway, University of Limerick, Maynooth University and Atlantic Technological University). The top performer in the IrMO is awarded the *Fergus Gaines cup*; congratulations to Fionn Kimber O'Shea, who achieved this honour in IrMO 2023.

The students who participate in the IrMO usually attend extra-curricular Mathematics Enrichment classes, which are offered at the six Mathematics Enrichment Centres listed in the previous paragraph. These classes run each year from January until April and are offered by volunteer academic mathematicians from these universities or nearby third-level institutions. More information on the organisation of these classes, as well as links to the individual maths enrichment centres, can be found at the Irish Maths Enrichment/IrMO website <http://www.irmo.ie/>.

Name	School	Year
Fionn Kimber O'Shea	Christian Brothers College, Wellington Road, Cork	5 <sup>th</sup>
Tianci Yan	Wesley College, Ballinteer, Dublin 16	3 <sup>rd</sup>
Emily Wolfe	Bruce College, St. Patrick's Hill, Cork	6 <sup>th</sup>
Seán Hallissey	Clongowes Wood College, Clane, Co. Kildare	6 <sup>th</sup>
Yuan Li	Sandford Park School, Ranelagh, Dublin 6	5 <sup>th</sup>
Owen Barron	Coláiste an Spioraid Naoimh, Bishopstown, Cork	3 <sup>rd</sup>

Table 1: The Irish contestants at the 64<sup>th</sup> IMO

The selection and training for IMO 2023 followed procedures which are by now well-established. First, an Irish Maths Olympiad “Squad” was identified, consisting of the top performers in IrMO 2022 who were eligible to qualify for the Irish IMO team in 2023. For these students, a kick-start training camp was held at UCC from 24–27 August 2022; such training camps are very important, as during these mathematically intense 3–5 day events, students have the opportunity to socialise with their peers, exchange their mathematical ideas, and increase their motivation for their work throughout the year. The kick-start training camp was organised by Anca Mustața, and classes were conducted by Eugene Gath, Bernd Kreussler, Declan Manning, Anca Mustața, Andrei Mustața and Andrew Smith, as well as 2022 Irish IMO team members Fionn Kimber O’Shea, Rory Moore, Taiga Murray and Emily Wolfe.

Between the end of the kick-start training camp and the beginning of the 2023 Mathematics Enrichment classes, the members of the Irish IMO Squad participated in the “remote training” programme, which operates as follows. At the beginning of each month from September to December inclusive, two sets of three problems are emailed to the participating students. They return their (complete or incomplete) solutions, by email or by post, to the proposer of the problems before the end of the month. The problem proposer then provides feedback on their work, as well as full solutions. This programme is very important for the successful engagement of “returning” students, and helps to develop the students’ independence in mathematical problem solving. In 2022, 27 students comprised the Irish IMO Squad, and the trainers involved in the remote training were Mark Flanagan, Eugene Gath, Gordon Lessells, Richard Magambo, Evariste Migabo, Anca Mustața, Andrei Mustața, Prasanna Ramakrishnan and Harun Šiljak.

Each year in November, the Irish Mathematical Olympiad starts with *IrMO Round 1*, a contest that is held in schools during a regular class period. In 2022, more than 13,000 students, mostly in their senior cycle, participated in Round 1. Teachers were encouraged to hand out invitations to their best performing students to attend the mathematics enrichment classes in their nearest mathematics enrichment centre.

Having participated in other Mathematical Olympiads before is an advantage for students when they participate in the IMO. In the last decade, more opportunities to do so have been created for members of the Irish Maths Olympiad Squad. In the current year, in addition to the possibility to compete in the European Girls’ Mathematical Olympiad (EGMO), which is for female students only, the members of the Irish Maths Olympiad Squad were invited to participate in the 2022 Iranian Combinatorics Olympiad, at which Fionn Kimber O’Shea, Emily Wolfe and Ayushi Mahajan won a Gold medal and 2nd place overall, and in the Iranian Geometry Olympiad (14 October 2022) at which Justin Li won a Bronze medal in the Intermediate category. The Irish Squad members were also invited to participate in the British Mathematical Olympiad Round 1 (16 November 2022), in which Fionn Kimber O’Shea was awarded a Gold medal and Seán Hallissey was awarded a Bronze medal, and a selection of the Irish IMO Squad were invited to participate in the British Mathematical Olympiad Round 2 (25 January 2023). Thanks to the organisers of the Iranian geometry and combinatorics olympiads, and to the UKMT (and in particular Geoff Smith), for giving our students these valuable opportunities.

For all students who participate in national mathematics enrichment classes, not only for the Squad members, a problem-solving contest was run simultaneously in each of the six maths enrichment centres on 25 February 2023. The results of this contest were used to select the team to represent Ireland at EGMO 2023.

Two further training camps were also organised at various locations shortly in advance

of IMO 2023. A training camp for the top performing students in IrMO 2023 was held at Mary Immaculate College, Limerick, from 7–9 June 2023, featuring an IMO-style exam in which  $3\frac{1}{2}$  hours were given to solve 3 problems. A pre-IMO training camp was also held for the members of the Irish IMO team, which took place in the Hotel Cadenza, Nerima, Tokyo from 2–6 July 2023. These training camps were organised by Bernd Kreussler and Mark Flanagan respectively, with generous help from Yui Murray in organising the training camp in Tokyo. The training sessions at the camps were conducted by Mark Flanagan, Ronan Flatley, Luke Gardiner, Eugene Gath, Bernd Kreussler, Gordon Lessells, Taiga Murray, Anca Mustăța and Andrew Smith.

## 2 The days in Chiba

The Irish IMO team, together with the Leader and Deputy Leader, arrived in Tokyo, Japan, on the morning of Saturday the 1<sup>st</sup> of July. This was the venue of the team's final intensive pre-IMO training camp (2–6 July 2023). This camp was held at the Hotel Cadenza in Nerima, where we were delighted to find that the hotel premises were excellently equipped for the training camp activities. Also, the hotel staff were assiduous in ensuring that we were provided with everything that we needed for the training. One additional trainer was available for the training sessions in addition to the Leader and Deputy Leader, namely Taiga Murray, a previous Irish contestant at IMO 2022. These training sessions consisted of many hours of intensive problem-solving, covering all four of the IMO contest topics. On July 3, I left Luke and the students at the training camp and travelled to the Jury site.

The Jury of the IMO, which is composed of the Team Leaders of the participating countries and a Chairperson who is appointed by the organisers, is the prime decision making body for all IMO matters. This year the IMO Jury was chaired by Takahiko Fujita with help from co-chairs Yongjin Song and Genki Shimizu; together they did an excellent job of making sure that all of the Jury meetings ran smoothly and efficiently. The Jury's most important task is choosing the six contest problems out of a shortlist of 30 problems provided by the IMO Problem Selection Committee, also appointed by the host country. While serving on the Jury, the Team Leaders are sequestered in a location sufficiently distant from that of the team and are forbidden from communicating with the team or Deputy Leader until the end of the second day of examinations. After many lively discussions at the Jury meetings, we had no issues in choosing six beautiful problems for the IMO 2023 papers.

For the eleventh year in a row, a problem selection protocol was followed whereby one problem from each of the four areas (algebra, combinatorics, geometry and number theory) would be included in problems 1, 2, 4 and 5. This protocol has the principal advantage of ensuring a balance between the four areas among the less difficult problems in the contest. As is now standard procedure at the IMO, electronic voting machines were used during the Jury meetings, ensuring both efficiency and anonymity in voting procedures.

After the exams are concluded, the final marks for each contestant are agreed in a process known as coordination. This important part of the IMO is well-established and ensures that the scripts of the students from so many different nations are marked fairly and consistently. The decisions in this process are based on detailed and strict marking schemes prepared by the coordination teams.

The marking of the scripts of each participating country is undertaken by two indepen-

dent groups. One group consists of the Team Leader, the Deputy Leader and the Official Observers (if any). The second group consists of the coordinators, who are appointed by the local organisers. This is an intensive task for the Leader and Deputy Leader, as we need to have a full understanding of the solution or attempts of each of our six students so that we can explain the merits of the students' work to the coordinators.

On looking through the students' scripts, it soon became clear that we had a strong Irish team performance on our hands. In particular, Fionn completely solved Problems 1, 4 and 5, giving him the fourth-highest score in Irish IMO history. Fionn's achievement of solving two full problems on Day 2 was particularly impressive, since these problems (a tricky inequality, and a challenging combinatorics problem on "Japanese triangles") were generally found to be harder than the corresponding problems on Day 1. Also, Emily and Seán completely solved Problem 4 and Problem 1 (a number theory problem) respectively. In a symmetrical fashion, Emily and Seán also presented very nice near-solutions to Problems 1 and 4 respectively, in each case requiring only a minor additional step in order to solve the problem completely. In general, both myself and Luke were very impressed by the array of creative mathematical ideas on display in the students' scripts (not all of which were eventually rewarded by the marking schemes). That some of these creative ideas were demonstrated by our youngest team members was particularly encouraging.

During the two days of coordination, excursions and other activities were organised for the students. Our students enjoyed an exciting trip to Tokyo Disneyland on the 10<sup>th</sup> of July, as well as visits to the "Skytree" of Tokyo Solamachi and to the Sensoji Temple in Asakusa on the 11<sup>th</sup> of July.

The final Jury meeting, at which the medal cut-offs were decided, took place on Tuesday 11<sup>th</sup> July. At this meeting, an irregularity was reported: at the end of one of the exams, one student continued to write their solutions even after being repeatedly told to stop doing so. The Jury decided to set the score for this student's best question on that day to zero.

The closing ceremony was held on Wednesday, 12<sup>th</sup> July, followed by a farewell party that evening featuring music and dancing. The team returned to Ireland on 13<sup>th</sup> July.

### 3 The problems

The two exams took place on the 8<sup>th</sup> and 9<sup>th</sup> of July, starting at 9:00am each morning. On each day,  $4\frac{1}{2}$  hours were available to solve three problems.

#### First Day

**Problem 1.** Determine all composite integers  $n > 1$  that satisfy the following property: if  $d_1, d_2, \dots, d_k$  are all the positive divisors of  $n$  with  $1 = d_1 < d_2 < \dots < d_k = n$ , then  $d_i$  divides  $d_{i+1} + d_{i+2}$  for every  $1 \leq i \leq k - 2$ .

(Colombia)

**Problem 2.** Let  $ABC$  be an acute-angled triangle with  $AB < AC$ . Let  $\Omega$  be the circumcircle of  $ABC$ . Let  $S$  be the midpoint of the arc  $CB$  of  $\Omega$  containing  $A$ . The perpendicular from  $A$  to  $BC$  meets  $BS$  at  $D$  and meets  $\Omega$  again at  $E \neq A$ . The line through  $D$  parallel to  $BC$  meets line  $BE$  at  $L$ . Denote the circumcircle of triangle  $BDL$

by  $\omega$ . Let  $\omega$  meet  $\Omega$  again at  $P \neq B$ .

Prove that the line tangent to  $\omega$  at  $P$  meets line  $BS$  on the internal angle bisector of  $\angle BAC$ .

(Portugal)

**Problem 3.** For each integer  $k \geq 2$ , determine all infinite sequences of positive integers  $a_1, a_2, \dots$  for which there exists a polynomial  $P$  of the form  $P(x) = x^k + c_{k-1}x^{k-1} + \dots + c_1x + c_0$ , where  $c_0, c_1, \dots, c_{k-1}$  are non-negative integers, such that

$$P(a_n) = a_{n+1}a_{n+2} \cdots a_{n+k}$$

for every integer  $n \geq 1$ .

(Malaysia)

## Second Day

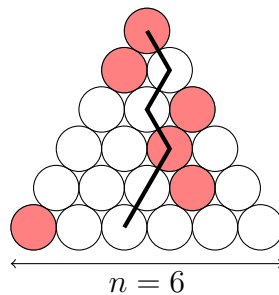
**Problem 4.** Let  $x_1, x_2, \dots, x_{2023}$  be pairwise different positive real numbers such that

$$a_n = \sqrt{(x_1 + x_2 + \dots + x_n) \left( \frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right)}$$

is an integer for every  $n = 1, 2, \dots, 2023$ . Prove that  $a_{2023} \geq 3034$ .

(Netherlands)

**Problem 5.** Let  $n$  be a positive integer. A *Japanese triangle* consists of  $1 + 2 + \dots + n$  circles arranged in an equilateral triangular shape such that for each  $i = 1, 2, \dots, n$ , the  $i^{\text{th}}$  row contains exactly  $i$  circles, exactly one of which is coloured red. A *ninja path* in a Japanese triangle is a sequence of  $n$  circles obtained by starting in the top row, then repeatedly going from a circle to one of the two circles immediately below it and finishing in the bottom row. Here is an example of a Japanese triangle with  $n = 6$ , along with a ninja path in that triangle containing two red circles.



In terms of  $n$ , find the greatest  $k$  such that in each Japanese triangle there is a ninja path containing at least  $k$  red circles.

(Netherlands)

**Problem 6.** Let  $ABC$  be an equilateral triangle. Let  $A_1, B_1, C_1$  be interior points of  $ABC$  such that  $BA_1 = A_1C$ ,  $CB_1 = B_1A$ ,  $AC_1 = C_1B$ , and

$$\angle BA_1C + \angle CB_1A + \angle AC_1B = 480^\circ.$$

Let  $BC_1$  and  $CB_1$  meet at  $A_2$ , let  $CA_1$  and  $AC_1$  meet at  $B_2$ , and let  $AB_1$  and  $BA_1$  meet at  $C_2$ .

Prove that if triangle  $A_1B_1C_1$  is scalene, then the three circumcircles of triangles  $AA_1A_2$ ,  $BB_1B_2$  and  $CC_1C_2$  all pass through two common points.

(Note: a scalene triangle is one where no two sides have equal length.)

(USA)

## 4 The results

The Jury tries to choose the contest problems such that Problems 1 and 4 are the most accessible, while Problems 2 and 5 are more challenging. Problems 3 and 6 are usually the most difficult problems, whose existence on the paper is justified in posing a sizeable challenge even to the top students in the IMO competition. Table 2, which shows the scores achieved by all contestants on the 6 problems, illustrates that this gradient of difficulty was generally maintained this year also.

	P1	P2	P3	P4	P5	P6
0	26	202	396	86	219	555
1	19	100	102	100	29	11
2	67	6	7	32	174	36
3	9	62	23	8	52	4
4	9	20	8	4	4	1
5	6	7	6	1	13	1
6	8	6	3	3	9	4
7	474	215	73	384	118	6
average	5.845	3.162	1.256	4.717	2.417	0.275

Table 2: The number of contestants achieving each possible number of points on Problems 1–6

The medal cut-offs were as follows: 32 points needed for a Gold medal (54 students), 25 for Silver (90 students) and 18 for Bronze (170 students). A further 192 students were awarded an Honourable Mention (an Honourable Mention is awarded to any student who did not win a medal, but achieved 7 points out of 7 on at least one problem).

From the available statistical information it can be noted that the scores at IMO 2023 were in general somewhat higher than at other recent IMO contests (although lower than those of last year’s IMO). In particular, the scores for Problem 1 were noticeably higher than usual. In total, 42.1% of all possible points were scored at this year’s IMO; this is only the seventh time since Ireland’s first participation in 1988 that the total lies above 40%. Having said this, it is worth pointing out that this year’s IMO contest still presented a significant challenge to all contestants, as can especially be seen from the low average scores on Problems 3 and 6.

Table 3 shows the results of the Irish contestants. The performance of the Irish team this year was a strong one, with one Bronze medal and two Honourable Mentions. The team scored a total of 77 points, the sixth best score of an Irish team at the IMO. The standout result is that of Fionn Kimber O’Shea, who achieved the fourth-highest score ever obtained by an Irish contestant at the IMO. Also, Emily Wolfe and Seán Hallissey were awarded Honourable Mentions for their complete solutions to Problems 4 and 1, respectively.

Name	P1	P2	P3	P4	P5	P6	total	ranking
Fionn Kimber O'Shea	7	0	0	7	7	0	21	237
Seán Hallissey	7	0	0	4	2	0	13	407
Emily Wolfe	2	0	0	7	2	0	11	425
Tianci Yan	2	4	0	1	2	0	9	459
Owen Barron	1	0	0	3	2	0	6	515
Yuan Li	2	0	0	1	0	0	3	535

Table 3: The results of the Irish contestants

The figures in Table 4 have the following meaning. The first figure after the topic indicates the percentage of all points scored out of the maximum possible. The second number is the same for the Irish team and the final column indicates the Irish average score as a percentage of the overall average. It can be seen from the table that the established trend of a strong Irish team performance in combinatorics was continued this year also, while some work could be done to strengthen Irish IMO performance in the areas of number theory and geometry.

The detailed results can be found on the official IMO website, which is located at <https://www.imo-official.org>.

Problem	topic	all countries	Ireland	relative
1	number theory	83.5	50.0	59.9
2	geometry	45.2	9.5	21.1
3	algebra	17.9	0.0	0.0
4	algebra	67.4	54.8	81.3
5	combinatorics	34.5	35.7	103.4
6	geometry	3.9	0.0	0.0
all		42.1	25.0	59.4

Table 4: Relative results of the Irish team for each problem

## 5 Outlook

The next countries to host the IMO will be

- 2024 United Kingdom
- 2025 Australia
- 2026 China
- 2027 Hungary

It is worth noting that IMO 2024 was originally scheduled to take place in Ukraine. However, due to the uncertainty regarding the viability of holding an IMO in Ukraine in 2024, a vote was held among the Leaders of the regularly participating countries to determine whether (due to *force majeure*) the hosting of IMO 2024 should be transferred to the United Kingdom in order to ensure the continuity of the IMO. As a result, the host country of IMO 2024 was changed to the United Kingdom. Ukraine has priority in organizing the next available IMO after 2024 and after the war, as soon as it is ready to do so.

## 6 Conclusions

This year's IMO featured a strong Irish team performance. Fionn Kimber O'Shea achieved a Bronze medal, while Emily Wolfe and Seán Hallissey were each awarded an Honourable Mention. Also, in general many innovative mathematical approaches were brought by the students to tackle the problems.

It is notable that Ireland's performance in Mathematical Olympiads has become generally stronger and more consistent in the past decade. This increased performance is underpinned by training activities which have grown in scope and impact. Of particular importance is the successful roll-out of the Junior Maths Enrichment (JME) programme, consisting of mathematical problem-solving activities for Junior Cycle students, within four universities across Ireland: University College Cork, University College Dublin, University of Galway, and Maynooth University. The JME programme is very important in that it engages Irish students in mathematical problem-solving activities at an earlier age; it is well-known that such students have a greatly enhanced probability to succeed at a high level. Feedback from students and parents on the JME programme continues to be overwhelmingly positive.

Of course, the delivery of the new Maths Enrichment activities described above, as well as the running of training camps and the sending of a full team of six students, together with Leader and Deputy Leader, to the IMO contest requires sustained funding. It is of primary importance that sufficient funding becomes available for these activities. An increased level of funding would also allow the scope of these initiatives to be widened further, so that the performance of Irish students in international mathematics contests can continue to improve year on year. Additional funding would also allow consolidation of the practice of sending an Irish Observer to the IMO, which has always been found to be a very beneficial practice.

## 7 Acknowledgements

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- University of Galway:  
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- University of Limerick:  
Mark Burke, Ronan Flatley, Mary Frawley, Sarah Frawley, Eugene Gath, Bernd Kreussler, Gordon Lessells, Edin Omerdic.
- Maynooth University:  
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